



## Cambridge International AS & A Level

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

### **FURTHER MATHEMATICS**

9231/32

Paper 3 Further Mechanics

October/November 2024

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

#### **INSTRUCTIONS**

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use  $10 \,\mathrm{m\,s^{-2}}$ .

#### **INFORMATION**

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has 16 pages. Any blank pages are indicated.

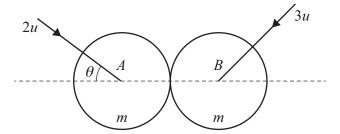
A particle of mass 2 kg is attached to one end of a light elastic string of natural length 0.8 m and modulus of elasticity 100 N. The other end of the string is attached to a fixed point O on a smooth horizontal surface. The particle is moving in a horizontal circle about O with the string taut and with constant angular speed 5 radians per second.
Find the extension of the string. [3]

A particle P of mass m is attached to one end of a light elastic spring of natural length a and modulus of elasticity 5mg. The other end of the spring is attached to a fixed point O. The spring hangs vertically with P below O. The particle P is pulled down vertically and released from rest when the length of the spring is  $\frac{3}{2}a$ .

3

Find the distance of $P$ below $O$ when $P$ first comes to instantaneous rest.	[4]
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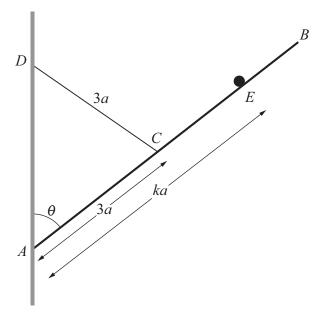


The diagram shows two identical smooth uniform spheres A and B of equal radii and each of mass m. The two spheres are moving on a smooth horizontal surface when they collide with speeds 2u and 3u respectively. Immediately before the collision, A's direction of motion makes an angle  $\theta$  with the line of centres and B's direction of motion is perpendicular to that of A. After the collision, B moves perpendicular to the line of centres. The coefficient of restitution between the spheres is  $\frac{1}{3}$ .

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∥           (b)	Find the total loss of kinetic energy as a result of the collision.
(~)	
(c)	Find, in degrees, the angle through which the direction of motion of $A$ is deflected as a result of collision.





6

The end A of a uniform rod AB of length 6a and weight W is in contact with a rough vertical wall. One end of a light inextensible string of length 3a is attached to the midpoint C of the rod. The other end of the string is attached to a point D on the wall, vertically above A. The rod is in equilibrium when the angle between the rod and the wall is  $\theta$ , where  $\tan \theta = \frac{3}{2}$ . A particle of weight W is attached to the point E on the rod, where the distance AE is equal to ka (3 < k < 6) (see diagram). The rod and the string are in a vertical plane perpendicular to the wall. The coefficient of friction between the rod and the wall is  $\frac{1}{3}$ . The rod is about to slip down the wall.

(a)	Find the value of $k$ .	[5]

* (	7
(b)	Find, in terms of $W$ , the magnitude of the frictional force between the rod and the wall. [2]
_	

A particle P is projected from a point O on horizontal ground with speed u at an angle  $\theta$  above the horizontal, where  $\tan \theta = \frac{1}{3}$ . The particle P moves freely under gravity and passes through the point with coordinates  $(3a, \frac{4}{5}a)$  relative to horizontal and vertical axes through O in the plane of the motion.

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At the instant when P is moving horizontally, a particle Q is projected from O with speed V at an angle  $\alpha$  above the horizontal. The particles P and Q reach the ground at the same point and at the same time.

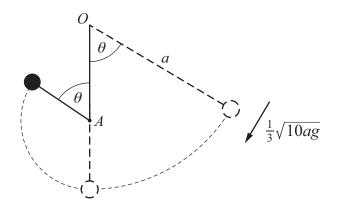
1	Express $V^2$ in the form $kag$ , where $k$ is a rational number.
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[6]



(a) Find the value of  $\cos \theta$ .

10



A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle P is held with the string taut and the string makes an angle  $\theta$  with the downward vertical through O. The particle P is projected at right angles to the string with speed  $\frac{1}{3}\sqrt{10ag}$  and begins to move downwards along a circular path. When the string is vertical, it strikes a small smooth peg at the point A which is vertically below O. The circular path and the point A are in the same vertical plane. After the string strikes the peg, the particle P begins to move in a vertical circle with centre A. When the string makes an angle  $\theta$  with the upward vertical through A the string becomes slack (see diagram). The distance of A below O is  $\frac{5}{9}a$ .

(b)	Find the ratio of the tensions in the string immediately before and immediately after it strikes th peg. [4

7	A particle $P$ of mass $m \log i$ is held at rest at a point $O$ and released so that it moves vertically under gravity against a resistive force of magnitude $0.1mv^2 N$ , where $v \ln s^{-1}$ is the velocity of $P$ at time $t s$ .
	gravity against a resistive force of magnitude 0.1mv 14, where viirs — is the velocity of T at time ts.

(a)	Find an expression for $v$ in terms of $t$ .	[6]
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The displacement of P from O at time ts is x m.

Find an expression for $v^2$ in terms of $x$ .

13

# Additional page

14

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